Electron Nonequilibrium in Open-Cycle MHD Generators

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Localized regions close to the electrode walls of open-cycle MHD generators have high enough current density so that the free electrons are not in thermal equilibrium with the rest of the gas, even when the working fluid is such an effective absorber of electron energy as CO_2 or coal combustion products. The over-all electrical performance of open-cycle MHD generators depends critically on the presence and behavior of such thin non-equilibrium regions adjacent to the electrodes. These effects are demonstrated and analyzed by means of previously developed numerical modeling techniques.

I. Introduction

OPEN-CYCLE magnetohydrodynamic (MHD) generators are customarily viewed as devices in which the electrons are in thermal equilibrium with the heavy components of the gas. Accordingly, the term "equilibrium generators" is often used when referring to such generators. The present paper examines whether electron equilibrium is indeed achieved throughout the geometry of such generators. It is shown that there are always thin localized regions close to the electrode walls, where the electrons are not in thermal equilibrium with the gas, even when the working gas is an extremely effective absorber of electron energy, such as coal combustion products; the term "equilibrium generator" can be applied with validity only to the core of the flow, where there are no current concentrations; at the electrode "hot spots," where the current concentrates and the gas temperature may be considerably lower than in the core of the flow, the electron temperature elevation in such generators can reach several hundred degrees Kelvin. The effect of these localized nonequilibrium regions on the electrical performance of open-cycle, combustion-gas-driven, MHD generator channels is assessed in this paper by the numerical modeling methods described in Ref. 1.

Before we present the results of the detailed numerical modeling, however, it is useful to give a simple estimate of the elevation of the electron temperature T_e above the characteristic temperature of the gas T in collision-dominated plasmas, and to define a characteristic parameter, f, that can be used for order-of-magnitude estimates. For this purpose, we shall employ the electron energy equation in the simplified form of an algebraic balance between the local Joule dissipation in the gas $\mathbf{E}'\cdot\mathbf{J}$ and the rate of energy transfer between the free electrons and the heavy particles $R_e^{(2)} \equiv \frac{3}{2}kn_e v_t \delta_{\text{eff}}(T_e - T)$. This results, in the first approximation,[2] in the simple expression

$$(T_e - T)/T = (\gamma/3) f (1-K_\infty)^2 M_\infty^{\ 2} \beta_\infty^{\ 2}/\lambda_{\text{eff}} \qquad (1)$$

This simplified expression, used here only for the purpose of defining the characteristic dimensionless quantity f and illustrating its significance, does not account for chemical reaction and assumes perfect gas behavior. It is valid at any point in the gas, provided that the factor f is defined locally as

$$f \equiv (T_\infty/T)(n_{e,\infty}^{\ 2}/n_e^{\ 2})(J^2/J_\infty^{\ 2})$$
$$\equiv (T_\infty/T)\left[(1+\beta_\infty^{\ -2})/(1+\beta^{-2})\right](E'^2/E_\infty'^2) \qquad (2)$$

In Eq. (1), γ is the ratio of the specified heats of the gas, $K \equiv E_y/UB$ is the Faraday loading parameter, M is the Mach number, $\beta \equiv \omega\tau$ is the local Hall parameter, and λ_{eff} is the ratio of the electron energy-loss of the gas δ_{eff} to the elastic value $\delta_{\text{eff,el}} = 2m_e/\langle m_G \rangle \equiv 1/(911W)$, where $\langle m_G \rangle$ is the mean weight of each gas particle and W the molecular weight of the gas. In Eq. (2), n_e is the electron number density and E'^2 means $|\mathbf{E}'|^2$, where $\mathbf{E}' \equiv \mathbf{E} + \mathbf{U}\times\mathbf{B}$ is the electric field in the moving gas. The conditions at the core of the flow have been indicated by the subscript ∞. It is clear that in the core of the flow the factor f is identically equal to 1. On the other hand, near the electrode surfaces, the factor f can have very high values, especially in channels with cold walls, because of the current concentration, which can make J/J_∞ much higher than 1, and because of the lower gas temperature near the wall compared to the core ($T/T_\infty < 1$), that also tends to keep $T_e/T_{e,\infty}$ and $n_e/n_{e,\infty}$ lower than 1 in open-cycle generators. The second form of Eq. (2) shows that the factor f will be very closely equal to the ratio of the electric field intensities $E'^2/E_\infty'^2$, and can therefore be called the "field intensity factor."

The elementary description given by Eq. (1) makes it clear that in the core of open-cycle, combustion-gas-driven MHD generators, the electrons will be in thermal equilibrium with the characteristic temperature of the gas T. Indeed, in such generators the core Mach number is seldom higher than 2, the core Hall parameter seldom higher than 4, δ_{eff} is of the order of 3×10^{-2} compared to the elastic value 3.66×10^{-5} for $W = 30$, so that $\lambda_{\text{eff}} \approx 10^3$, and therefore, for $K_\infty = 0.5$–0.7, the electron temperature elevation in the core of the flow is found to be negligible indeed:

$$[(T_e - T)/T]_{\text{core}} \approx 10^{-3}$$

Therefore, the presence of electron nonequilibrium is limited to regions where the electric field intensity factor f becomes very large, for example 100. Such values of f occur naturally near the surface of segmented electrode walls, where the magnitude of $\mathbf{E}'$ can easily be one order of magnitude higher than $|\mathbf{E}_\infty'|$. The effect of these nonequilibrium regions, which are always present, on the over-all electrical performance and efficiency of open-cycle MHD generators is assessed in the following by means of detailed numerical modeling. For this purpose, we have utilized the rigorous and complete formulation of the electrical problem in MHD generators (that includes nonuniformities, convective relaxation effects, finite reaction rates, etc.) and the numerical methods of solution that have been developed previously.[3-5] All results presented in the following have been obtained by these rigorous methods.

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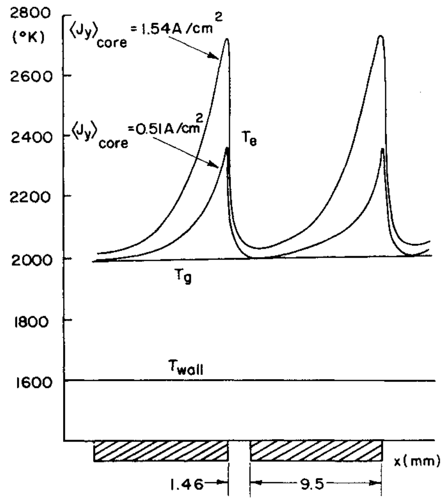

Fig. 1 Axial variation of the electron temperature at the edge of the laminar sublayer on the cathode wall of a combustion-gas-driven MHD generator: $U_\infty = 1775$ m/sec, $T_\infty = 2670$°K, $p = 1.15$ atm, $B = 3$ tesla.

II. Effects of Electron Nonequilibrium on Local and Over-All Electrical Performance

The electrode wall region of MHD generators is characterized by severe nonuniformities in all three classes of variables discussed in Ref. 1: the current density and electric field distributions; the plasma property distributions; and the gasdynamic velocity, temperature, and density distributions. They are the result of the field discontinuities at the conductor-insulator intersections of their segmented walls, and of the steep gasdynamic profiles, especially when the generator is running with cold walls.

The effect of electron nonequilibrium on the over-all performance is coupled to these nonuniformities in a nonlinear fashion that is fully accounted for by the numerical modeling.

The computations presented in this paper refer to typical open-cycle conditions, namely a generator running with the combustion products of coal burned with stoichiometric air and expanded to the following core conditions: gas velocity 1775 m/sec, temperature 2670°K, pressure 1.15 atm, and a magnetic field of 3 tesla. The properties of this working fluid have been

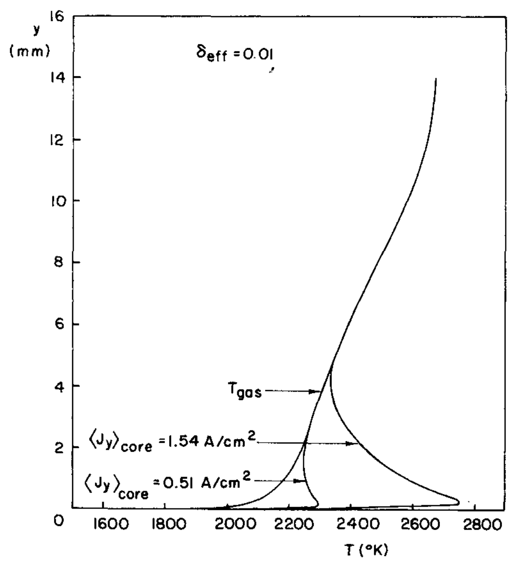

Fig. 2 Electron temperature profile across the boundary layer as a function of average current density $\langle J_y \rangle$ in the core for a given gas temperature profile.

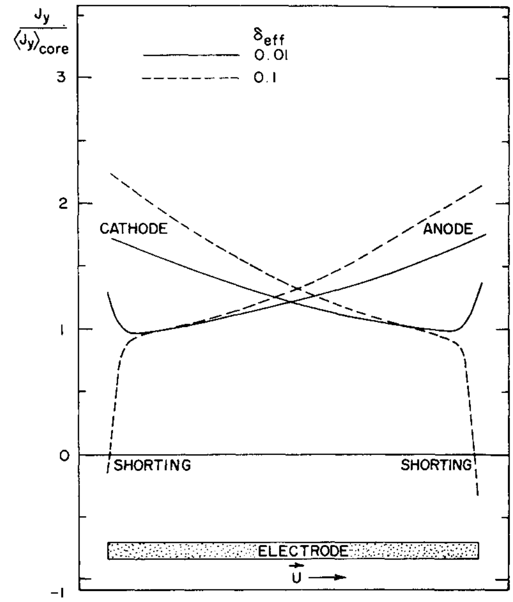

Fig. 3 The influence of electron nonequilibrium on the current distribution on the cathode surface of a combustion-gas-driven MHD generator (Avco Mark VI). Note the switch from shorting to more even distribution as δ_{eff} decreases and electron nonequilibrium increases.

computed by the methods of Ref. 6. For the purposes of this example, the channel is assumed to have smooth walls.

The results of the numerical modeling concerning the electron nonequilibrium near the walls under these conditions are shown in Figs. 1 and 2. These figures show that the electron temperature elevation above the gas temperature can amount to several hundred degrees Kelvin near the electrode "hot spots," and that these elevations increase with the average current density in the core. Figure 3 shows the current density distribution on the surface of the conductor for different values of the electron energy-loss factor of the gas. The lower value $\delta_{eff} = 0.01$, that allows higher electron nonequilibrium, leads to a more uniform distribution of current over the cold conductor surface, which may at first seem unexpected. The reason is that lower δ_{eff} allows the electron temperature (upon which the electrical conductivity depends) to rise above the gas temperature (which has a sharp gradient in the vicinity of a cold wall) and thus results in smaller transverse nonuniformities of the electrical conductivity that have been shown[1,4,7] to control the local behavior close to the walls and the over-all electrical performance.

The effect that electron nonequilibrium near the electrode walls has on the over-all electrical performance of this channel is illustrated by Fig. 4, which shows the electrical output power density P/V, as a function of mean current density in the core of the flow, for different values of the electron energy-loss factor of the gas, δ_{eff}. When δ_{eff} increases from 0.01 to 0.1, there results a 17% reduction of the maximum obtainable power density P/V in this generator, which occurs at approximately $\langle \mathbf{J} \rangle_{core} =$ 1 A/cm² when $\delta_{eff} = 0.01$, and 0.8 A/cm² when $\delta_{eff} = 0.1$. At higher current densities, such as 2 A/cm², the electrode voltage drops are too high for this channel to behave as a generator; additional external voltage and power would be then needed to pass so much current, and the device would act as a "decelerator" of the gas through application of external fields. Figure 4 also shows the effect of electron nonequilibrium on the loading parameters K and $\langle K \rangle$ defined as

$$K = (E_y)_{core}/U_{core}B$$

$$\langle K \rangle = \int_0^D E_y \, dy / U_{core} BD$$

where D is the height of the channel. It is well known[1] that in terms of K and $\langle K \rangle$, the electrical power density $P/V \equiv \langle \mathbf{E} \cdot \mathbf{J} \rangle$ can be written as

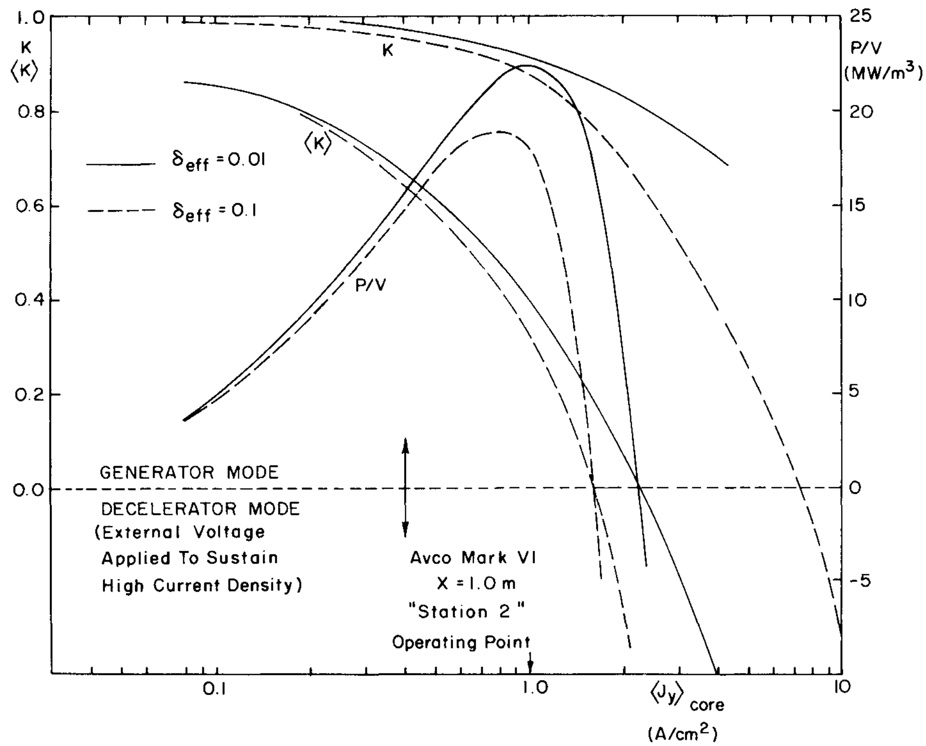

Fig. 4 The influence of electron nonequilibrium on the over-all performance characteristics of a combustion-gas-driven MHD generator (Avco Mark VI). Performance is evaluated assuming electron energy-loss factors $\delta_{\text{eff}} = 0.01$ and 0.1.

$$P/V = (\sigma/\varepsilon)_{\text{eff}} U_{\text{core}}^2 B^2 \langle K \rangle (1-K)$$

where

$$(\sigma/\varepsilon)_{\text{eff}} \equiv (J_y)_{\text{core}}/(UB - E_y)_{\text{core}}$$

The average loading parameter $\langle K \rangle$ is a measure of the Faraday potential.

III. Conclusion

The over-all electrical performance of open-cycle MHD generators is governed by the local behavior in a very thin region close to the electrode walls.[8] The presence of electron nonequilibrium in this region has a beneficial effect on the over-all electrical performance of the generator, although possibly not on its longevity. For average current densities below 0.5 A/cm^2, this effect is rather small. However, for current densities of the order of 2 A/cm^2, such as may be used for peaking plant, emergency, and other applications, electron nonequilibrium can improve the electrical performance significantly. On the other hand, for these high average current densities, electron nonequilibrium decreases the critical Hall parameter for ionization instabilities,[4] and may thus introduce such instabilities in the generator[9]; this factor has not been taken into consideration in the abovementioned sample computations. It also increases the possibility of other inherently unsteady phenomena that have not been considered here, such as arcing and breakdown across insulator segments.

References

[1] Demetriades, S. T., Argyropoulos, G. S., and Maxwell, C. D., "Progress in Analytical Modeling of MHD Power Generators," Paper I.5, 12th Symposium on Engineering Aspects of Magnetohydrodynamics, 1972, Argonne National Lab., Argonne, Ill.

[2] Demetriades, S. T. and Argyropoulos, G. S., "Ohm's Law in Multicomponent Nonisothermal Plasmas with Temperature and Pressure Gradients," *The Physics of Fluids*, Vol. 9, 1966, pp. 2136–2149.

[3] Argyropoulos, G. S., Demetriades, S. T., and Kendig, A. P., "Current Distribution in Nonequilibrium $J \times B$ Devices," *Journal of Applied Physics*, Vol. 38, 1967, pp. 5233–5239.

[4] Argyropoulos, G. S. and Demetriades, S. T., "Influence of Relaxation Effects in Nonequilibrium $J \times B$ Devices," *Journal of Applied Physics*, Vol. 40, Oct. 1969, pp. 4400–4409.

[5] Argyropoulos, G. S., Casteel, M. A., and Demetriades, S. T., "Two-Dimensional Distribution of Current Along Magnetohydrodynamic Channels," *Energy Conversion*, Vol. 10, 1970, pp. 189–192.

[6] Maxwell, C. D., Demetriades, S. T., Argyropoulos, G. S., Patel, N. J., and Easterling, M., "Input Data for Computation of Thermodynamic and Electrical Properties of Coal Combustion Products," Paper VII.5, 13th Symposium on Engineering Aspects of Magnetohydrodynamics, 1973, Stanford, Calif.

[7] Rosa, R. J., "Hall and Ion-slip Effects in a Nonuniform Gas," *The Physics of Fluids*, Vol. 5, 1962, pp. 1081–1090.

[8] Hsu, M. S. S., "Thermal Instabilities and Arcs in the MHD Boundary Layers," Paper VI.6, 13th Symposium on Engineering Aspects of Magnetohydrodynamics, 1973, Stanford, Calif.

[9] Solbes, A., "Instabilities in MHD Plasmas, A Review," AIAA Paper 70-40, New York, 1970.